# Electrical Circuits (2)

## Lecture 7
## Transient Analysis

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# Extra Reference for this Lecture
# Chapter 16

Schaum's Outline Of Theory And Problems Of Electric Circuits

https://archive.org/details/TheoryAndProblemsOfElectricCircuits

# Circuits Transient Response

- When a circuit is switched from one condition to another either by a change in the applied voltage or a change in one of the circuit elements, there is a transitional period during which the branch currents and voltage drops change from their former values to new ones

- After this transition interval called the transient, the circuit is said to be in the steady state.

- So far all the calculations we have performed have led to a Steady State solution to a problem i.e. the final value after everything has settled down.

- Transient analysis: study of circuit behavior in transition phase.

- The steady state values can be determined using circuit laws and complex number theory.
- The transient is more difficult as it involves differential equations.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_0 x(t) = f(t)$$

- General solution to the differential equation:

$$x(t) = x_p(t) + x_c(t)$$

$x_p(t)$

- Particular integral solution (or forced response particular to a given source/excitation)
- Represent the steady-state solution which is the solution to the above non-homogeneous equation

$x_c(t)$

- Complementary solution (or natural response)
- Represent the transient part of the solution, which is the solution of the next homogeneous equation:

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_0 x(t) = 0$$

## First-Order and Second-Order Circuits

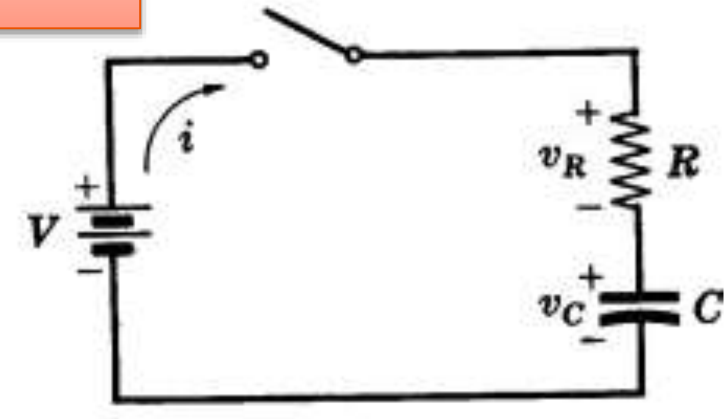
- First-order circuits contain only a single capacitor or inductor
- Second-order circuits contain both a capacitor and an inductor

## Differential equations Solutions

- Two techniques for transient analysis that we will learn:
  - ✓ Differential equation approach.
  - ✓ Laplace Transform approach.

- Laplace transform method is a much simpler method for transient analysis but we will see both ☺ :P

# First-Order RC Transient Step-Response




- Assume the switch S is closed at t = 0
- Apply KVL to the series RC circuit shown:

$$\frac{1}{C}\int i\,dt + Ri = V$$

- Differentiating both sides which gives:

$$\frac{i}{C} + R\frac{di}{dt} = 0 \quad \text{or} \quad \left(D + \frac{1}{RC}\right)i = 0$$

- The solution to this homogeneous equation consists of only the complementary function since the particular solution is zero.
- To find the complementary Solution, solve the auxiliary equation:

$$m + \frac{1}{RC} = 0 \qquad m = \frac{-1}{RC} = \frac{-1}{\tau} \qquad \tau = RC$$

Time constant

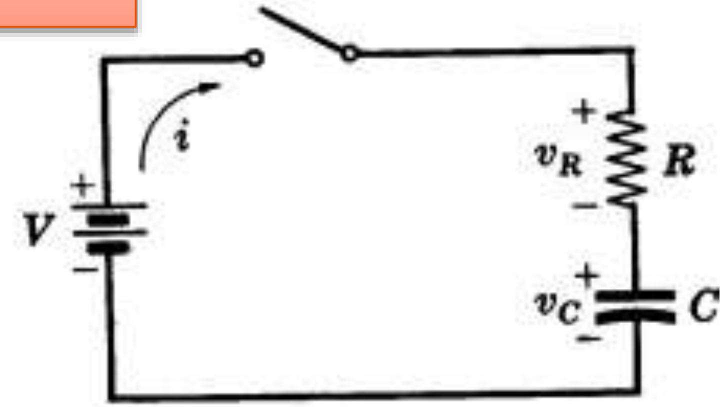
The complementary Solution is :

$$i = Ae^{mt} \qquad i = Ae^{\frac{-t}{\tau}}$$

# First-Order RC Transient Step-Response

- To determine the constant "A" we note that :

at $t = 0$ is $Ri_0 = V$ or $i_0 = V/R.$

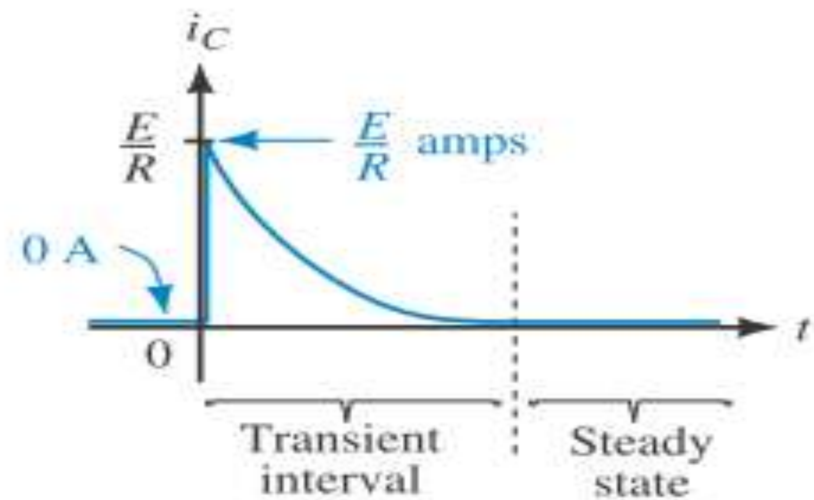



Where Vc (0) = 0

- Now substituting the value of $i_o$ into current equation
- We obtain A = V/R at t = 0.

$$i = \frac{V}{R} e^{-t/RC}$$

has the form of an exponential decay starting from the transient value to the final steady-state value of 0 ampere in 5 time-constants



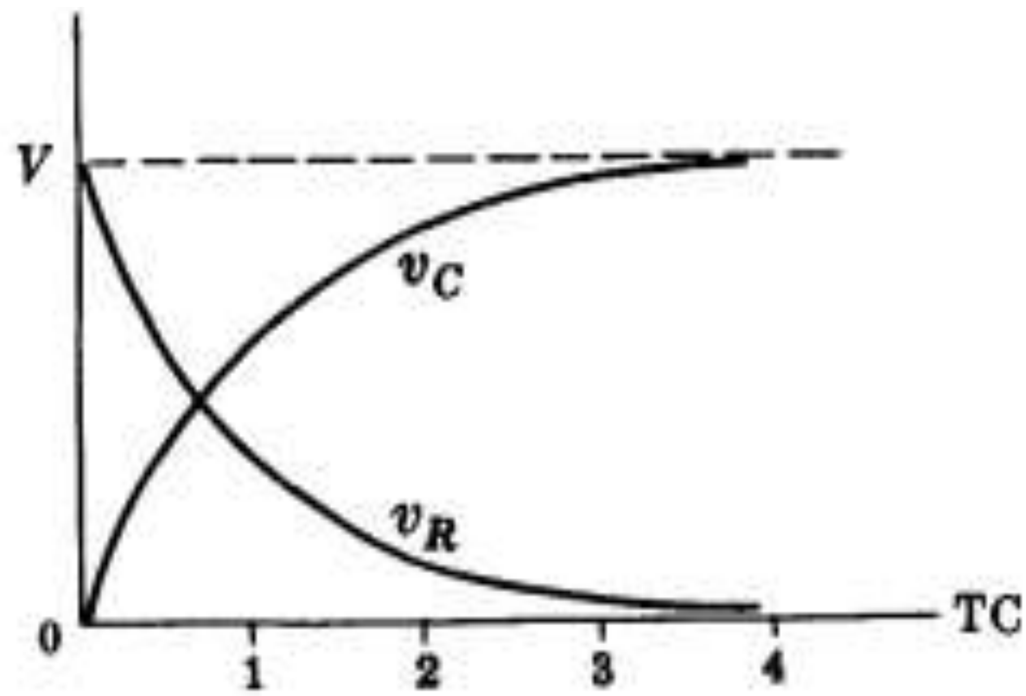
# First-Order RC Transient Step-Response

- The voltage across the resistor is:

$$v_R = Ri = Ve^{-t/RC}$$

- The voltage across the capacitor is:

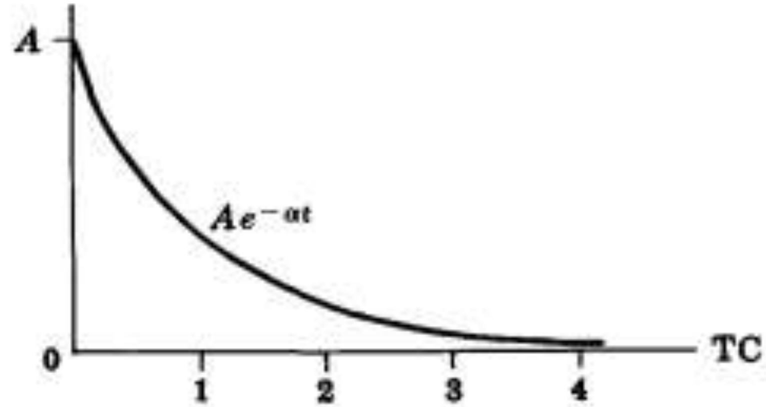
$$v_c = \frac{1}{C}\int i\,dt = V(1 - e^{-t/RC})$$



# Time-Constant

Transient-response is almost finished after 5τ

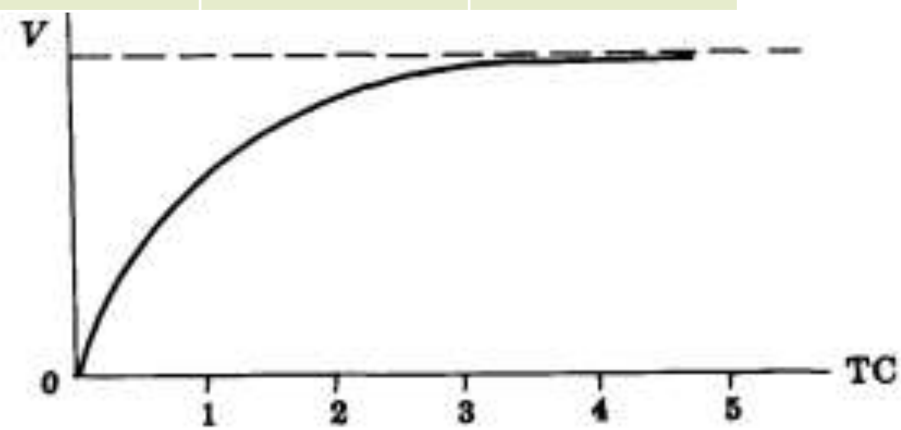
## 1. Exponential-Decay




| t | τ | 2τ | 3τ | 4τ | 5τ |
|---|---|---|---|---|---|
| i(t) | $0.368\,\frac{V}{R}$ | $0.135\frac{V}{R}$ | $0.05\frac{V}{R}$ | $0.018\frac{V}{R}$ | $0.007\frac{V}{R}$ |

## 2. Exponential-Rise

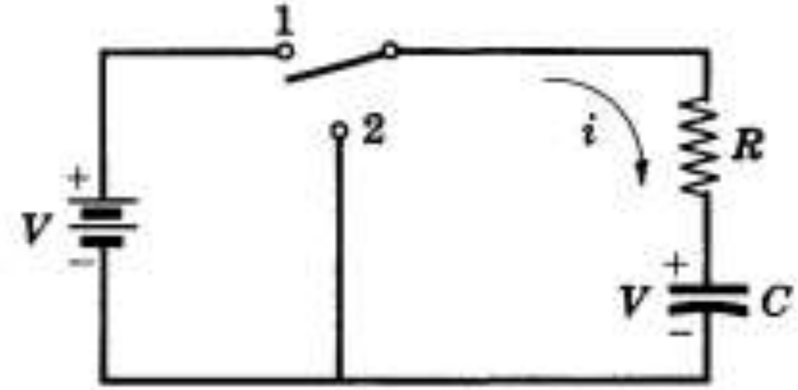
| t | τ | 2τ | 3τ | 4τ | 5τ |
|---|---|---|---|---|---|
| Vc(t) | $0.632\ V$ | $0.865\ V$ | 0.95 V | 0.98 V | 0.99 V |



# First-Order RC Transient (Discharge)



- The series RC circuit shown in Figure has the switch in position 1 for sufficient time to establish the steady state
- At  t = 0,  the switch is moved to position 2




$$\frac{1}{C}\int i\,dt + Ri = 0$$

- Differentiating both sides  which gives:

$$\frac{i}{C} + R\frac{di}{dt} = 0 \quad \text{or} \quad \left(D + \frac{1}{RC}\right)i = 0$$

The Solution also  is :

$$i = Ae^{mt} = Ae^{\frac{-t}{RC}}$$

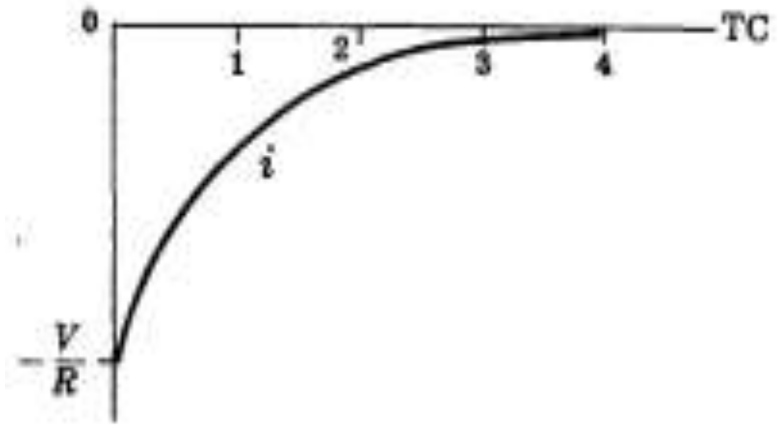
- Substitute by the initial condition of the current to get the constant A:
- Since the capacitor is charged to a voltage V with the polarity shown in the diagram, the initial current is opposite to i;

$i_0 = -V/R.$  Then  $A = -V/R$  Then

$$i = (-V/R)e^{\frac{-t}{RC}}$$

# First-Order RC Transient (Discharge)

- The decay transient of the current is shown in figure

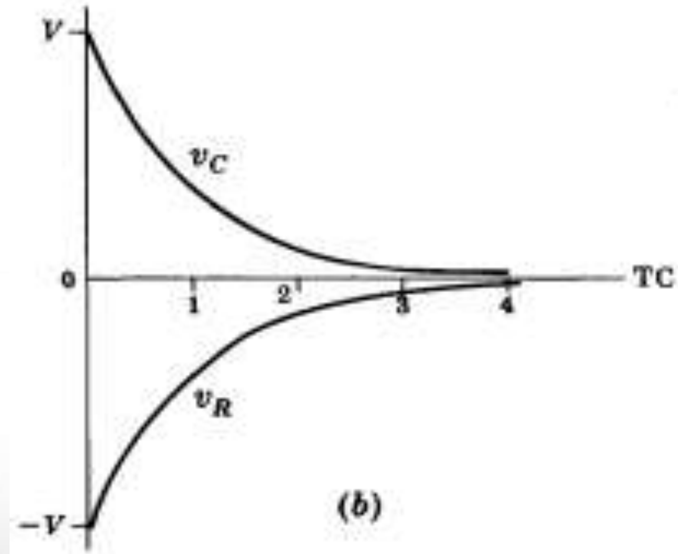



- The corresponding transient voltages

$$v_R = Ri = -Ve^{-t/RC}$$

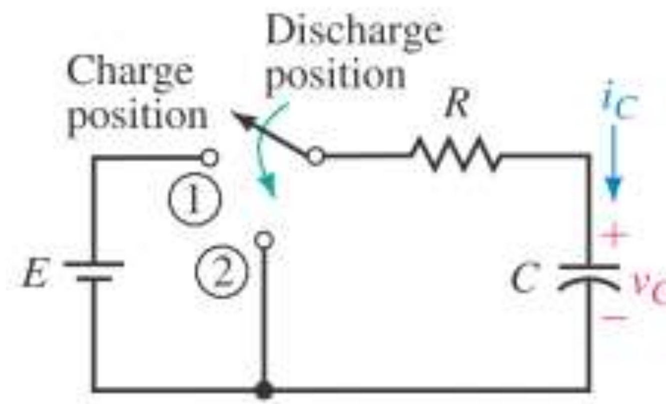
$$v_C = \frac{1}{C}\int i\,dt = V\,e^{-t/RC}$$

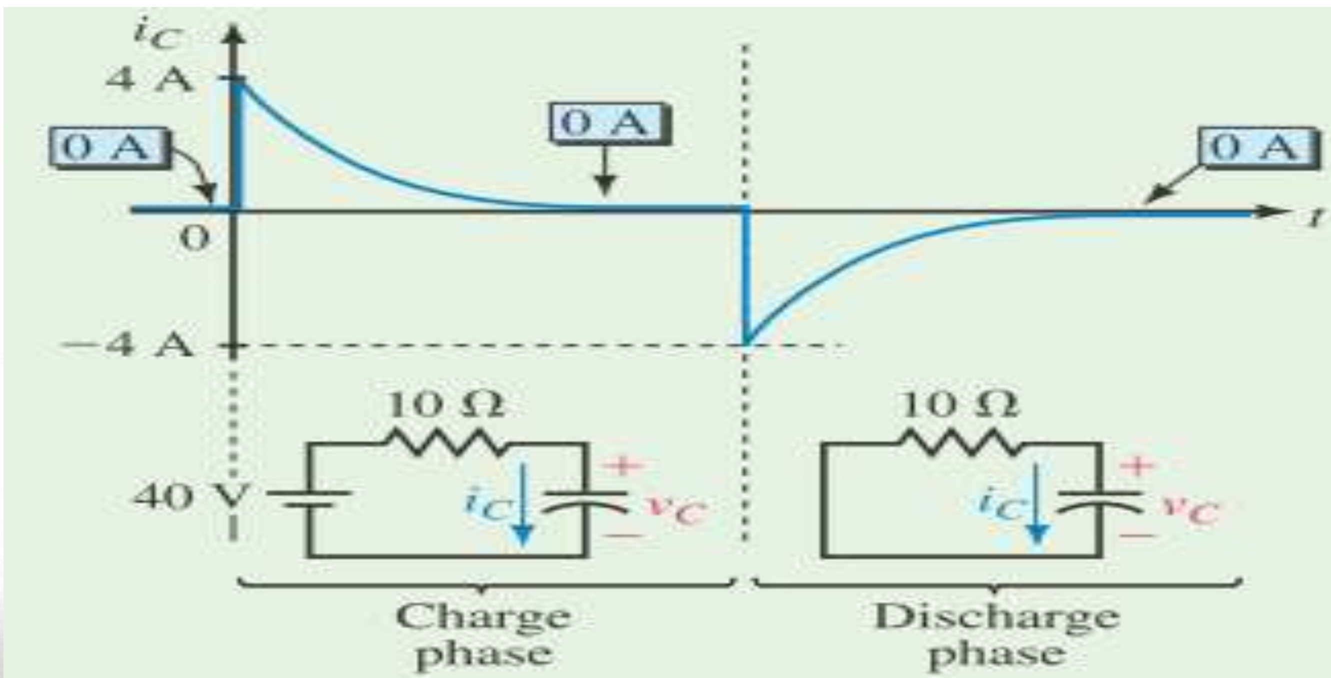
Note that $v_R + v_C = 0$



Examples

For Figure 11–1, $E = 40$ V, $R = 10\ \Omega$, and the capacitor is initially uncharged. The switch is moved to the charge position and the capacitor allowed to charge fully. Then the switch is moved to the discharge position and the capacitor allowed to discharge fully. Sketch the voltages and currents and determine the values at switching and in steady state.

# Capacitor with an Initial Voltage

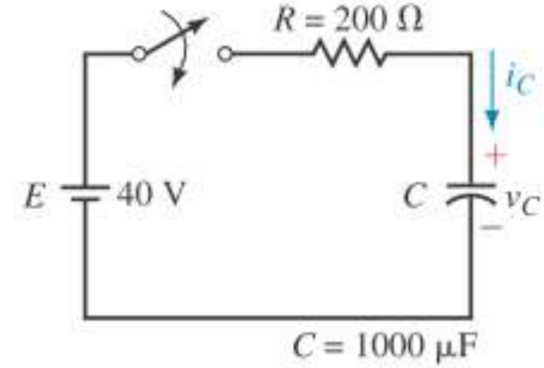


- Suppose a previously charged capacitor has not been discharged and thus still has voltage on it.

$$v_C = E + (V_0 - E)e^{-t/\tau}$$

$$i_C = \frac{E - V_0}{R} e^{-t/\tau}$$

Ex: Suppose the capacitor of Figure 11–16 has 25 volts on it with polarity shown at the time the switch is closed.

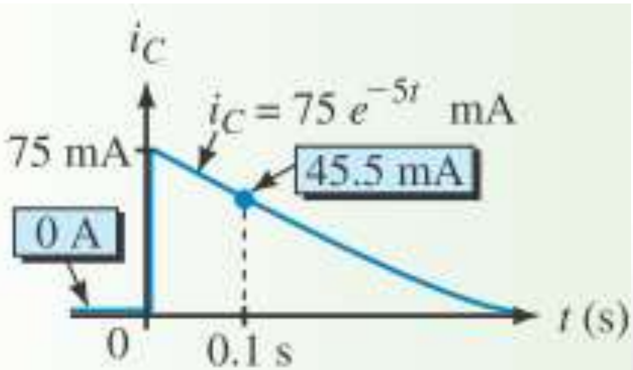


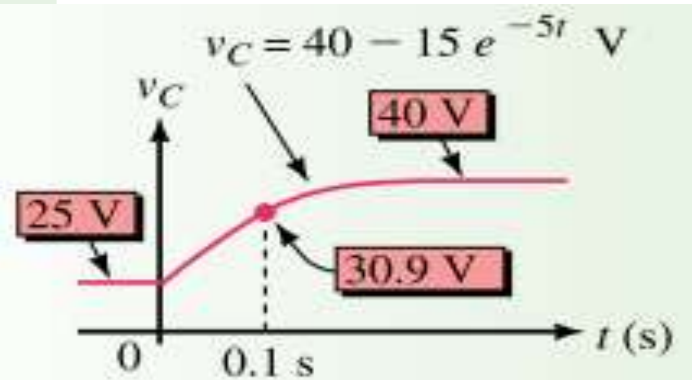

**Solution** $\tau = RC = (200\ \Omega)(1000\ \mu\text{F}) = 0.2\ \text{s}$

a. From Equation 11–10,

$$v_C = E + (V_0 - E)e^{-t/\tau} = 40 + (25 - 40)e^{-t/0.2} = 40 - 15e^{-5t}\ \text{V}$$

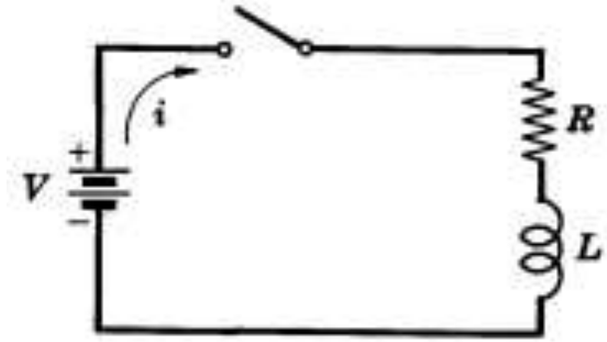
$$i_C = \frac{E - V_0}{R} e^{-t/\tau} = \frac{40 - 25}{200} e^{-5t} = 75e^{-5t}\ \text{mA}$$



# First-Order RL Transient Step-Response

- The switch "S" is closed at t = 0
- Apply KVL to the circuit in figure:




$$Ri + L\frac{di}{dt} = V$$

- Rearranging and using "D" operator notation :

$$\left(D + \frac{R}{L}\right)i = \frac{V}{L}$$

This Equation is a first order, linear differential equation

1. Complementary (Transient) Solution

The auxiliary equation is : $m + \frac{R}{L} = 0$

$$i = Ae^{mt} = Ae^{\frac{-R}{L}t}$$

$\tau = \frac{R}{L}$ Time constant

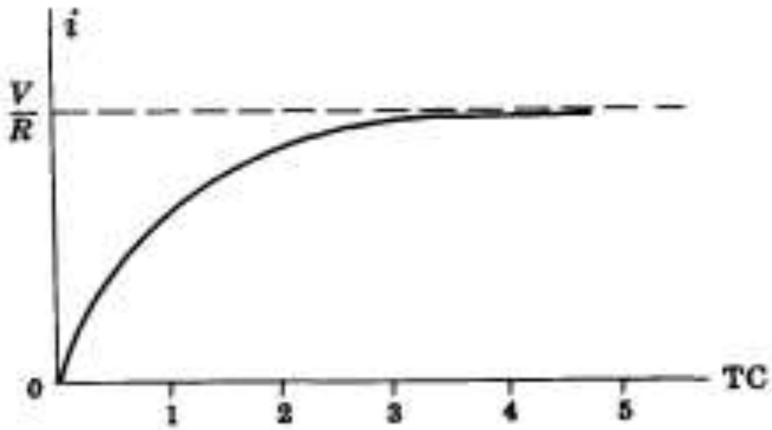
2. Particular (Steady-State) Solution

The steady-state value of the current for DC source is : $I_{ss} = \frac{V}{R}$

# First-Order RL Transient Step-Response

- The total solution is:

$$i = Ae^{\frac{-R}{L}t} + \frac{V}{R}$$



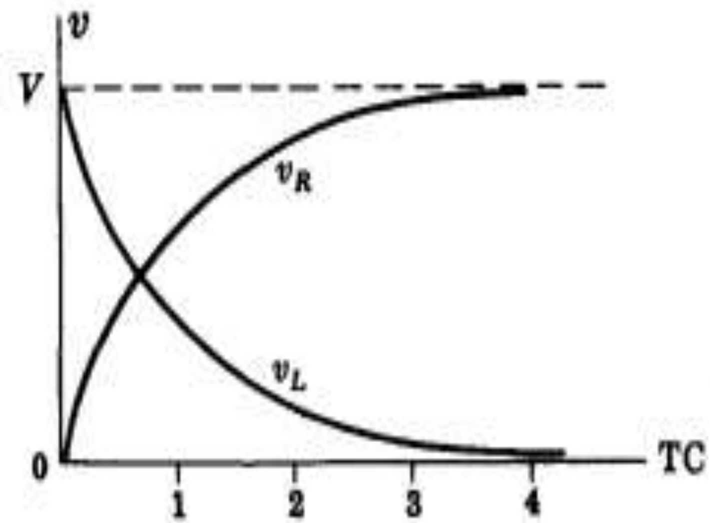

Since The initial current is zero:

$$0 = A + \frac{V}{R}$$

$$i = -\frac{V}{R}e^{-(R/L)t} + \frac{V}{R} = \frac{V}{R}(1 - e^{-(R/L)t})$$

- The voltage across the resistor is:

$$v_R = Ri = V(1 - e^{-(R/L)t})$$




- The voltage across the inductor is:

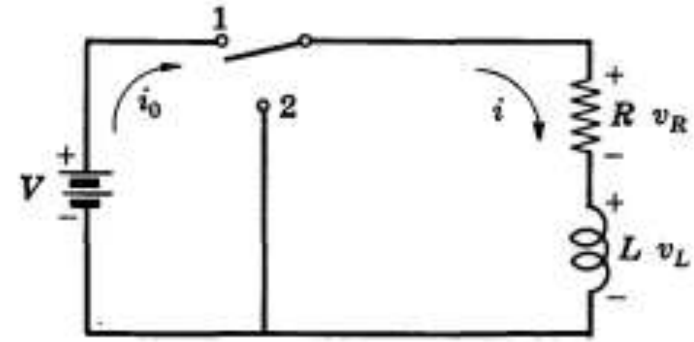
$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left\{\frac{V}{R}(1 - e^{-(R/L)t})\right\} = Ve^{-(R/L)t}$$

$$v_R + v_L = V(1 - e^{-(R/L)t}) + Ve^{-(R/L)t} = V$$

# First-Order RL Transient (Discharge)



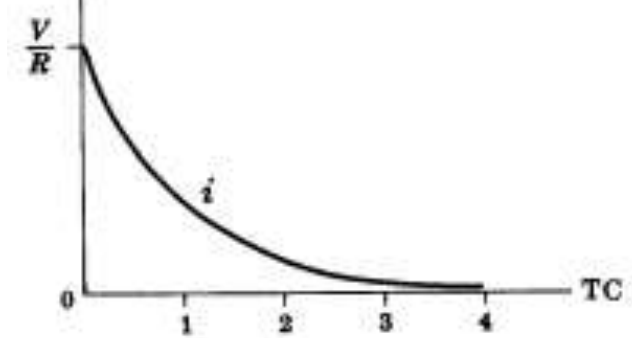
- The RL circuit shown in Figure contains an initial current of (V/R)
- The Switch "S" is moved to position"2" at t=0




$$L\frac{di}{dt} + Ri = 0 \quad \text{or} \quad \left(D + \frac{R}{L}\right)i = 0$$

- The solution is the transient (Complementary) part only.

$$i = ce^{-(R/L)t}$$



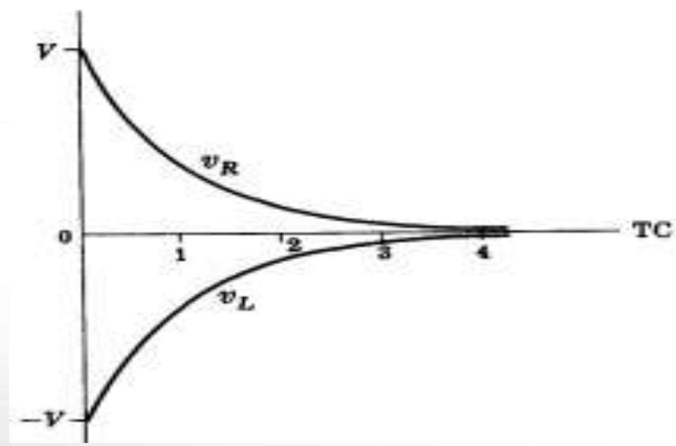

- Using the initial condition of the current, we get:

$$i = \frac{V}{R}e^{-(R/L)t}$$

- The corresponding voltages across the resistance and inductance are

$$v_R = Ri = Ve^{-(R/L)t}$$

$$v_L = L\frac{di}{dt} = -Ve^{-(R/L)t}$$



# Examples

A series $RL$ circuit with $R = 50$ ohms and $L = 10$ h has a constant voltage $V = 100$ v applied at $t = 0$ by the closing of a switch. Find (a) the equations for $i$, $v_R$ and $v_L$, (b) the current at $t = .5$ seconds and (c) the time at which $v_R = v_L$.

(a) The differential equation for the given circuit is

$$50i + 10\frac{di}{dt} = 100 \quad \text{or} \quad (D+5)i = 10$$

the complete solution is $i = i_c + i_p = ce^{-5t} + 2$

At $t = 0$, $i_0 = 0$ and $0 = c(1) + 2$ or $c = -2$. Then

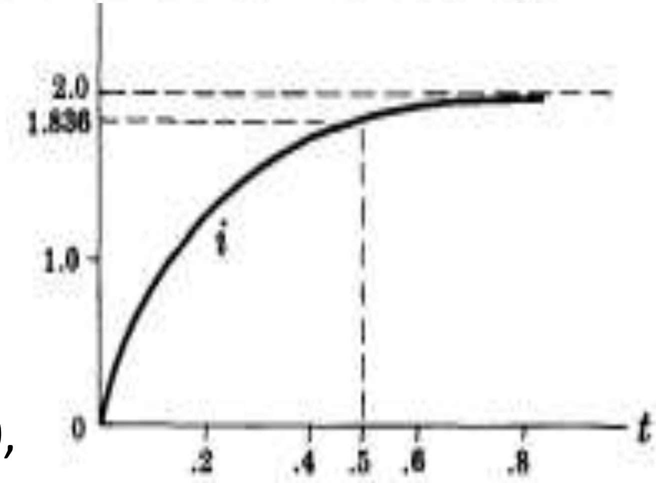
$$i = 2(1 - e^{-5t})$$

$$v_R = Ri = 100(1 - e^{-5t})$$

$$v_L = L\frac{di}{dt} = 100e^{-5t}$$

# Examples

(b) Put $t = .5$ sec in (3) and obtain $i = 2(1 - e^{-5(.5)}) = 2(1 - .082) = 1.836$ amp.




(b) For the two voltage to be equal:
each must be 50 volts since the applied voltage is 100,

$$v_L = 50 = 100e^{-5t},$$

$$e^{-5t} = .5 \quad \text{or} \quad 5t = .693,$$

$$t = .1386 \text{ sec.}$$